A Four-State Labelling Semantics for Weighted Argumentation Frameworks*

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Abstract

Computational Argumentation provides tools for both modelling and reasoning with controversial information. The building blocks in this field are represented by Abstract Argumentation Frameworks, namely structures which explicit the relationships between arguments in order to establish their acceptability. Indeed, arguments can be assigned different justification states: some of the arguments may be accepted, while some other rejected; it could also be the case that some arguments are ignored. Labels corresponding to such states are assigned through sets of criteria called labelling-based semantics. In this paper, we consider Weighted Argumentation Frameworks and propose a novel labelling-based semantics which differentiates four different states, also generalising existing approaches.

Keywords

Computational Argumentation, Weighted Abstract Argumentation Framework, Four-state Labelling

1. Introduction

Computational Argumentation and its applications are receiving increasing interest in many fields of AI. For instance, argumentative processes are used in a paper by Lawrence et al. [1] to interpret online debates, while Walton and Koszowy [2] devise an argumentation system for supporting expert opinion. Argumentation is also used to aid machine learning (as surveyed by Cocarascu and Toni [3]) for both improving performances (e.g., classification accuracy) and providing explanations for the results. Argumentation problems are modelled through Abstract Argumentation Frameworks (AFs in short) [4], which consist of directed graphs in which the nodes are arguments that contain abstract information and the edges represent attack relations. The main goal of these frameworks is to check the acceptability of arguments, which indicates how credible they can be when used, for example, in a speech or debate.

The acceptability of an argument of an AF can be established following different criteria, formalised through the extension-based [4] and the labelling-based semantics [5]. Through the reasoning on the acceptability of the arguments according to a notion of defence, one can divide the set of arguments into two separated subsets, respectively containing acceptable and

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non-acceptable arguments. Various approaches have been proposed to cope with the problem of detecting different justification states of arguments in AFs. Indeed, apart from accepted and rejected, arguments could be just ignored or even in an inconsistent state. Caminada [5], for example, introduces a labelling-based semantics in which the state of an argument can be left undecided, without further specifying the reason why. The motivation for not labelling an argument as neither accepted nor rejected is explicitly expressed by Jakobovits and Vermeir [6], who made a distinction between arguments we "don't care" about and those we "do not know" how to label.

In order to increase the expressiveness of AFs, attack relations between arguments can be endowed with a value (a weight) which indicates the strength of the attacks themselves. In this kind of frameworks, called Weighted AF, the acceptability criteria for the arguments also need to consider the weight of incoming and outgoing attacks. Bistarelli et al. [7, 8] group the attacks from an argument to a set of arguments as if they were a unique attack; in particular, the authors consider a weighted notion of defence that takes into account the weight associated with each attack, also generalising other approaches [9, 10].

In this paper, which complements a series of work [11, 12, 13, 14], we provide a four-state labelling for Weighted AFs that generalises other approaches proposed in the literature for the non-weighted case and the three-state labelling for Weighted AFs. For each weighted semantics, we give the conditions under which a labelling corresponds to an extension (that is a set of accepted arguments). We use a partial labelling (i.e., we can leave specific arguments unlabelled) with four labels to identify the possible states of arguments, namely `IN` for accepted, `OUT` for rejected, `DK` for arguments we don't know how to label, and `DC` for arguments we don't care about (because not adopted in an AF or just ignored by the user).

The rest of this paper is structured as follows. In Section 2 we summarise the main concepts of AFs, providing the definitions for extension-based semantics considering both weighted and non-weighted cases. In Section 3 we present our definition of four-state labelling for Weighted Argumentation Frameworks. Section 4 discusses relevant work on labelling-based semantics for (W)AFs already present in the literature, and finally, in Section 5 we conclude the paper, also discussing possible future research lines.

2. Preliminaries

In this section, we recall the formal definitions of AFs [4] and Weighted AFs [7, 8], together with te notion of extension- and labelling-based semantics [15, 5].

Definition 1 (Abstract Argumentation Framework). *Let $\mathcal{U}$ be the set of all available arguments[1]. An Abstract Argumentation Framework is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A} \subseteq \mathcal{U}$ is a set of arguments and $\mathcal{R}$ is a binary relation on $\mathcal{A}$. Arguments in $\mathcal{A}$ are said to be adopted.*

Definition 2 (Attacks). *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF, and consider two arguments $a, b \in \mathcal{A}$. If $(a, b) \in \mathcal{R}$, we say that a attacks b; conversely, b is an attacker of a. Moreover, given $A \subseteq \mathcal{A}$, we define*

[1] The set $\mathcal{U}$, which we refer to as the *Universe* of arguments, is not present in the original definition of AFs, and it is introduced to model arguments which are external to $\mathcal{A}$ [16, 17].

the sets $a^+ = \{b \in \mathcal{A} \mid (a,b) \in \mathcal{R}\}$, $a^- = \{b \in \mathcal{A} \mid (b,a) \in \mathcal{R}\}$, $A^+ = \bigcup\{a^+ \mid a \in A\}$ *and* $A^- = \bigcup\{a^- \mid a \in A\}$.

In order for an argument a to be acceptable, we require that every attacker of a is defeated in turn by some other argument.

Definition 3 (Acceptable argument). *Let* $\langle \mathcal{A}, \mathcal{R} \rangle$ *be an AF, and consider* $a \in \mathcal{A}$ *and* $D \subseteq \mathcal{A}$. *The argument* a *is acceptable with respect to the subset* D *if and only if* $\forall b \in A. \exists d \in D \mid (b \in a^-) \implies (d \in b^-)$. *In that case, we say that* a *is* ***defended*** *by* D *from the attack of* b.

We also say that argument is acceptable if there exists a subset of arguments with respect to which it is acceptable. Using the notion of defence as a criterion for distinguishing acceptable arguments in the framework, one can further refine the set of selected arguments through the so-called extension-based semantics.

Definition 4 (Extension-based semantics). *Given an AF* $\langle \mathcal{A}, \mathcal{R} \rangle$, *we say that a set of arguments* $E \subseteq \mathcal{A}$ *is conflict-free if and only if* $\nexists a, b \in E$ *such that* $(a,b) \in \mathcal{R}$. *A conflict-free set* E *is said to be*

- *admissible, if each* $a \in E$ *is defended by* E
- *complete, if it is admissible and* $\forall a \in \mathcal{A}$ *defended by* E, $a \in E$
- *stable, if* $E \cup E^+ = \mathcal{A}$
- *preferred, if it is complete and it is maximal (with respect to set inclusion)*
- *grounded, if it is complete and it is minimal (with respect to set inclusion)*

In this paper, we only consider the above semantics, although other extension-based semantics have also been defined in the literature, such as ideal, semi-stable and stage [15]. In Figure 1, we provide an example of an AF for which we compute the set S of conflict-free, admissible, complete, stable, preferred and grounded extensions (abbreviated with cf, adm, com, stb, prf and gde, respectively): $S_{cf}(F) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,d\}\}$, $S_{adm}(F) = \{\emptyset, \{a\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}\}$, $S_{com}(F) = \{\{a\}, \{a,c\}, \{a,d\}\}$, $S_{prf}(F) = \{\{a,c\}, \{a,d\}\}$, $S_{stb}(F) = \{\{a,d\}\}$ and $S_{gde}(F) = \{\{a\}\}$.

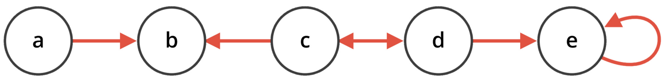

Figure 1: Example of an AF with five arguments.

We give some details on the extensions found. The singleton $\{e\}$ is not conflict-free because e attacks itself. Argument b is not contained in any admissible extension because no other argument (included itself) defends b from the attack of a. The empty set and the singletons $\{c\}$ and $\{d\}$ are not complete extensions because they do not contain a, which is not attacked by any other argument. Only the maximal complete extensions $\{a,c\}$ and $\{a,d\}$ are preferred, while the minimal complete $\{a\}$ is the unique grounded extension. Since argument a attacks arguments b and argument d attacks arguments c and e, we have that $\{a, d\}$ is a stable extension.

To obtain different nuances for the acceptability of arguments, we can rely on the notion of labelling-based semantics [5], namely functions that partitions the arguments of an AF into three subsets.

Definition 5 (Labelling for AFs). *Let $F = \langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A labelling L of F is a total function $L : \mathcal{A} \to \{\mathtt{IN}, \mathtt{OUT}, \mathtt{UNDEC}\}$.*

Notation 1. *Given a labelling L of $F = \langle \mathcal{A}, \mathcal{R} \rangle$ and $A \subseteq \mathcal{A}$, we denote $A \downarrow_{\mathtt{IN}}$, $A \downarrow_{\mathtt{OUT}}$ and $A \downarrow_{\mathtt{UNDEC}}$ the sets of all arguments labelled $\mathtt{IN}$, $\mathtt{OUT}$ and $\mathtt{UNDEC}$, respectively, by L.*

We show in Figure 2 an example of labelling: IN arguments are highlighted in green and OUT ones in red, while UNDEC are represented in yellow. It is also possible to identify a correspondence between labellings and sets of extensions for a certain semantics [15].

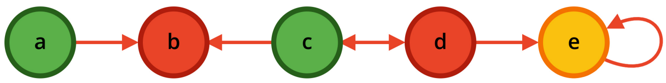

Figure 2: Example of labelling for an AF with five arguments.

Definition 6 (Labelling-based semantics). *Let L be a labelling of an AF $F = \langle \mathcal{A}, \mathcal{R} \rangle$ and $a \in \mathcal{A}$. Then*

- *L is a conflict-free labelling if:*
  - $L(a) = \mathtt{IN} \implies a^- \downarrow_{\mathtt{IN}} = \emptyset$, *and*
  - $L(a) = \mathtt{OUT} \implies a^- \downarrow_{\mathtt{IN}} \neq \emptyset$
- *L is a admissible labelling if:*
  - $L(a) = \mathtt{IN} \implies a^- = a^- \downarrow_{\mathtt{OUT}}$, *and*
  - $L(a) = \mathtt{OUT} \implies a^- \downarrow_{\mathtt{IN}} \neq \emptyset$
- *L is a complete labelling if:*
  - $L(a) = \mathtt{IN} \iff a^- = a^- \downarrow_{\mathtt{OUT}}$, *and*
  - $L(a) = \mathtt{OUT} \iff a^- \downarrow_{\mathtt{IN}} \neq \emptyset$
- *L is a stable labelling if:*
  - *L is a complete labelling, and*
  - $\mathcal{A} \downarrow_{\mathtt{UNDEC}} = \emptyset$;
- *L is a preferred labelling if:*
  - *L is an admissible labelling, and*
  - *$\mathcal{A} \downarrow_{\mathtt{IN}}$ is maximal among all the admissible labellings*

- *L is a grounded labelling if:*
  - *L is a complete labelling, and*
  - *$\mathcal{A}\downarrow_{IN}$ is minimal among all the complete labellings*

We have, for instance, that the labelling of Figure 2 is complete, but not grounded. Since all attacks in AFs have the same "strength", it is not possible to further diversify the relations among arguments, and thus the existence of an attack is the only thing that matters in determining the semantics. To overcome this limitation, we can resort to Weighted AFs, whose attacks are endowed with a value that represents the support of the relation [18]. In this kind of framework, the notion of defence needs to be adapted to encompass the refined attack relation. In a paper by Bistarelli et al. [8], Weighted AFs are equipped with a c-semiring [19, 20] that provides operations for composing the weights and estimating the effectiveness of a defence.

Definition 7 (c-semirings). *A c-semiring is a tuple $\mathbb{S} = \langle S, \oplus, \otimes, \bot, \top\rangle$ such that S is a set, $\top, \bot \in S$, and $\oplus, \otimes : S \times S \to S$ are binary operators making the triples $\langle S, \oplus, \bot\rangle$ and $\langle S, \otimes, \top\rangle$ commutative monoids (semi-groups with identity), satisfying* i) $\forall s, t, u \in S.\ s \otimes (t \oplus u) = (s \otimes t) \oplus (s \otimes u)$ *(distributivity), and* ii) $\forall s \in S.\ s \otimes \bot = \bot$ *(annihilator). Moreover, we have that $\forall s, t \in S.\ s \oplus (s \otimes t) = s$ (absorption). The operator $\oplus$ also defines a preference relation $\leq_{\mathbb{S}}$ over the set S, such that $a \leq_{\mathbb{S}} b$ if and only if $a \oplus b = b$, for all $a, b \in S$.*

We list some of the most common instances of c-semirings:

- $\mathbb{S}_{boolean} = \langle\{false, true\}, \vee, \wedge, false, true\rangle$
- $\mathbb{S}_{fuzzy} = \langle[0, 1], \max, \min, 0, 1\rangle$
- $\mathbb{S}_{probabilistic} = \langle[0, 1], \max, \times, 0, 1\rangle$
- $\mathbb{S}_{weighted} = \langle\mathbb{R}^+ \cup \{+\infty\}, min, +, +\infty, 0\rangle$

The interval $[0, 1]$ used for $\mathbb{S}_{fuzzy}$ and $\mathbb{S}_{probabilistic}$ is to be considered valid for both real and rational numbers. We denote with $\text{WAF}_{\mathbb{S}}$ a Weighted AF endowed with a c-semirings $\mathbb{S}$ and we call it a semiring-based Weighted AF.

Definition 8 (Semiring-based Weighted AF). *Let $\mathcal{U}$ be the set of all available arguments. A semiring-based Weighted AF is a quadruple $\langle\mathcal{A}, \mathcal{R}, W, \mathbb{S}\rangle$, where $\mathcal{A} \subseteq \mathcal{U}$ is the set of adopted arguments, $\mathcal{R}$ the attack relation on $\mathcal{A}$, $W : \mathcal{A} \times \mathcal{A} \to S$ a binary function, and $\mathbb{S}$ a c-semiring $\langle S, \oplus, \otimes, \bot, \top\rangle$.*

The binary function W assigns a weight to attacks between arguments: we use $W(a, b) = s$ to indicate that the attack from a towards b has weight $s \in S$. In our setting, the $\top$ element of a c-semiring (*e.g.*, 0 for the weighted and $true$ for the boolean) denotes the absence of a pair in the relation R. Hence, $(a, b) \in \mathcal{R}$ if and only if $W(a, b) <_{\mathbb{S}} \top$.

Given a $\text{WAF}_{\mathbb{S}}$, we can evaluate the overall weight of all the attacks from a set of arguments towards another set through the binary **composition** operator $\otimes$ of the c-semiring $\mathbb{S}$ [7, 21]. In particular, we use $\bigotimes$ to indicate the $\otimes$ operator on a set of values.

Definition 9 (Weighted attacks). *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a WAF$_{\mathbb{S}}$ and consider two sets of arguments $B, D \in \mathcal{A}$. We say that B attacks D, and the weight of such attack is $k \in S$, if*

$$W(B, D) = \bigotimes_{b \in B, d \in D} W(b, d) = k.$$

Following Definition 9, it is also possible to compose the attacks both from a set of arguments towards a single argument and from a single argument towards a set of arguments. We can now express the notion of weighted defence.

Definition 10 (Weighted defence). *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a WAF$_{\mathbb{S}}$. We say that $B \subseteq \mathcal{A}$ w-defends $b \in \mathcal{A}$ if and only if $\forall a \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$, $W(a, B \cup \{b\}) \geq_{\mathbb{S}} W(B, a)$.*

Consider the WAF$_{\mathbb{S}}$ of Figure 3. To verify whether the set $\{a\}$ w-defends d we need to check if $W(c, \{a, d\}) \geq_{\mathbb{S}} W(\{a\}, c)$. We have that $W(c, \{a, d\}) = 3$ and $W(\{a\}, c) = 2$, and since $3 \not\geq_{\mathbb{S}} 2$, we conclude that a alone is not sufficient to w-defend d in this example[2]. If we consider the set $\{a, b\}$, instead, we can see that $W(c, \{a, b, d\}) \geq_{\mathbb{S}} W(\{a, b\}, c)$ since $W(c, \{a, b, d\}) = W(\{a, b\}, c) = 3$, and therefore $\{a, b\}$ w-defends d.

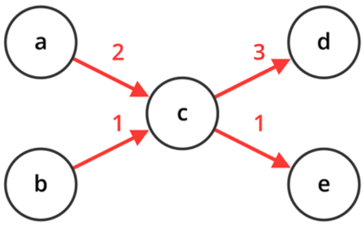

Figure 3: Example of a WAF$_{\mathbb{S}}$ with $\mathbb{S} = \mathbb{S}_{weighted}$.

Notation 2. *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a WAF$_{\mathbb{S}}$ and consider an argument $a \in \mathcal{A}$. We denote the weight of a set of attacks towards a with $w_{a^- \downarrow_{IN}} = W(a^- \downarrow_{IN}, a)$, and the weight of outgoing attacks with $w_{a^+ \downarrow_{IN}} = W(a, a^+ \downarrow_{IN})$.*

It is then possible to redefine all the extension-based semantics of Definition 4 by using the notion of weighted defence for checking the acceptability of arguments [8].

Definition 11 (Extension-based semantics for WAF$_{\mathbb{S}}$). *Consider a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and a subset of arguments $E \subseteq \mathcal{A}$. We have that E is w-conflict-free if $W(E, E) = \top$. A w-conflict-free subset E is*

- *w-admissible, if $\forall a \in E^-.\ W(a, E) \geq_{\mathbb{S}} W(E, a)$*
- *w-complete, if it is w-admissible and each $b \in \mathcal{A}$ such that $E \cup \{b\}$ is w-admissible belongs to E*
- *w-stable, if it is w-admissible and $\forall a \notin E.\ \exists b \in E$ such that $W(b, a) <_{\mathbb{S}} \top$*

[2]We remark that $3 <_{\mathbb{S}} 2$ when $\mathbb{S} = \mathbb{S}_{weighted}$, i.e., greater means worse.

- *w-preferred, if it is a maximal (with respect to set inclusion) w-admissible subset of $\mathcal{A}$*
- *w-grounded, if it is the maximal (with respect to set inclusion) w-admissible extension included in the intersection of w-complete extensions*

As for the non-weighted case, also sets of acceptable arguments in a $WAF_{\mathbb{S}}$ can be identified through special labelling functions. In the next section, we expand the discussion in this direction, introducing a weighted labelling that differentiates up to four states of acceptability.

3. From Three-State to Four-State Weighted Labelling

The labelling for AFs of Definition 5 and the derived labelling-based semantics are a useful tool which identifies up to three degrees of acceptability for the arguments while maintaining a direct connection with set of extensions for the classical semantics introduced by Dung [4]. However, the labelling function shown in the previous section forces all arguments that are neither `IN` nor `OUT` to be labelled `UNDEC`, thus not allowing to distinguish arguments we don't know how to label from arguments we deliberately decide to ignore. In other words, three labels are not sufficient to express the difference between the possible causes for which an argument can be labelled UNDEC. Consider for instance the AF in Figure 4, whose arguments are labelled according to the admissible labelling-based semantics. Arguments c and d are both labelled `UNDEC`, but for two distinct reasons: c, which could potentially be accepted (it has no `IN` attackers), is ignored, while d is attacking itself and thus it can neither be accepted nor rejected. To overcome these inconvenience, more informative labellings have been proposed [22, 6, 23] that split the `UNDEC` label into two distinct labels, resulting in a total of four recognisable acceptability states[3].

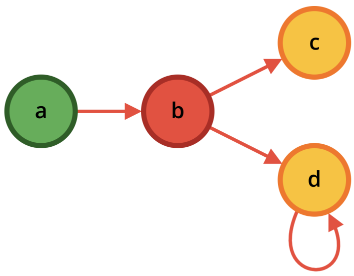

Figure 4: Example of labelling with two `UNDEC` arguments.

Before introducing our proposal for a labelling function able to work with $WAF_{\mathbb{S}}$ and which makes use of four labels, we recall the definition of three-state weighted labelling [11, 12, 14]. In order to incorporate the notion of weighted defence into the labelling, also the strength of the attack relations is taken into account.

Definition 12 (Three-state Labelling for $WAF_{\mathbb{S}}$). *Let $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ be a $WAF_{\mathbb{S}}$. A three-state labelling L of F is a total function $L : \mathcal{A} \to$ {`IN`, `OUT`, `UNDEC`}.*

[3] More nuances of acceptability can be enabled through ranking-based semantics [24], however, losing the correspondence with accepted arguments identified by extension-based semantics.

Definition 13 (Three-state labelling-based semantics for WAF$_{\mathbb{S}}$). *Consider a three-state labelling L of $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and an argument $a \in \mathcal{A}$.*

- *L is a w-conflict-free labelling when*
  - $L(a) = \mathtt{IN} \implies a^- \downarrow_{\mathtt{IN}} = \emptyset$ *and*
  - $L(a) = \mathtt{OUT} \implies a^- \downarrow_{\mathtt{IN}} \neq \emptyset$
- *L is a w-admissible labelling for F if and only if:*
  - $L(a) = \mathtt{IN} \implies a^- = a^- \downarrow_{\mathtt{OUT}} \wedge \forall b \in a^-.\, w_{b^- \downarrow_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+ \downarrow_{\mathtt{IN}}}$
  - $L(a) = \mathtt{OUT} \implies w_{a^- \downarrow_{\mathtt{IN}}} <_{\mathbb{S}} \top$
- *L is a w-complete labelling for F if and only if:*
  - $L(a) = \mathtt{IN} \iff a^- = a^- \downarrow_{\mathtt{OUT}} \wedge \forall b \in a^-.\, w_{b^- \downarrow_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^+ \downarrow_{\mathtt{IN}}}$
  - $L(a) = \mathtt{OUT} \iff w_{a^- \downarrow_{\mathtt{IN}}} <_{\mathbb{S}} \top$
- *L is a w-stable labelling for F if and only if*
  - *L is a w-complete labelling and*
  - $\mathcal{A} \downarrow_{\mathtt{UNDEC}} = \emptyset$
- *L is a w-preferred labelling for F if and only if*
  - *L is a w-admissible labelling and*
  - *$\mathcal{A} \downarrow_{\mathtt{IN}}$ is maximal among all the w-admissible labellings*
- *L is a w-grounded labelling for F if and only if:*
  - $L(a) = \mathtt{IN} \iff$ *for all w-complete labellings L', $L'(a) = \mathtt{IN}$ and*
  - $L(a) = \mathtt{OUT} \iff w_{a^- \downarrow_{\mathtt{IN}}} <_{\mathbb{S}} \top$

The sets of arguments labelled IN by the labelling-based semantics of Definition 13 are equivalent to extensions of the corresponding semantics. OUT and UNDEC arguments, instead, are considered to be rejected. Our proposal for a richer labelling function is based on four labels, namely IN, OUT, DK and DC.

Definition 14 (Four-State Labelling for WAF$_{\mathbb{S}}$). *Let $\mathcal{U}$ be a universe of arguments and $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ a WAF$_{\mathbb{S}}$ with $\mathcal{A} \subseteq \mathcal{U}$. A four-state labelling L of F is a partial function $L : \mathcal{U} \rightharpoonup \{\mathtt{IN}, \mathtt{OUT}, \mathtt{DK}, \mathtt{DC}\}$.*

Notation 3. *Given a four-state labelling L of $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$, $A \subseteq \mathcal{A}$ and $l \in \{\mathtt{IN}, \mathtt{OUT}, \mathtt{DK}, \mathtt{DC}\}$, we use $A \downarrow_l = \{a \in A \mid L(a) = l\}$ to restrict to arguments in A only labelled with l. We also denote with $L \downarrow_A$ a total mapping $L \downarrow_A : A \rightarrow \{\mathtt{IN}, \mathtt{OUT}, \mathtt{DK}, \mathtt{DC}\}$.*

We see in Figure 5 an example of four-state weighted labelling. Accepted and rejected arguments, labelled with IN and OUT as usual, are still highlighted in green and red, respectively. An argument with label DK, which is highlighted in yellow, could be both accepted and rejected, meaning that we cannot decide about its acceptability (we "don't know", indeed). The DC label is depicted in grey and stands for "don't care" [6] and identifies arguments that are not interesting

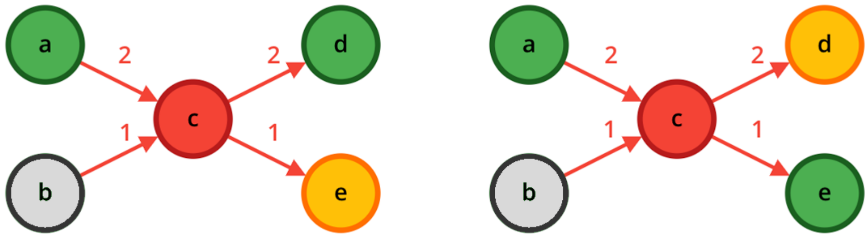

Figure 5: Two possible labellings of a WAF$_{\mathbb{S}}$ with $\mathbb{S} = \mathbb{S}_{weighted}$.

to analyse and that we just want to ignore. Finally, arguments in $\mathcal{U} \setminus \mathcal{A}$ (that are only part of the universe but not of the AF) are not labelled.

According to the definition of collective weighted defence (Definition 10), a set of arguments is defended from an attacker c only if the $\bigotimes$ of all the defending arguments is stronger than the $\bigotimes$ of the attacks coming from c. This means that the strength of the attacks of the defending arguments is distributed among the defended arguments and it is not guaranteed for two arguments that are separately w-defended to still be w-defended when considered together (this is what happens in the example of Figure 5 with arguments d and e).

We give a characterisation of four-state weighted semantics through the notion of labelling of WAF$_{\mathbb{S}}$ following the intuition that attacks of defending arguments are "consumed" by the defended one. In particular, an argument that cannot be accepted because its defenders are not strong enough will be labelled `UNDEC`. The first semantics we investigate is the basic requirement of conflict-freeness.

Fact 1 (w-conflict-free four-state labelling). *The w-conflict-free four-state labelling coincides with the w-conflict-free labelling.*

We want to identify a set of non-conflicting arguments, so we don't have to consider the weight of the attacks, but only if attacks exist between arguments in this set. We now define the w-admissible four-state labelling.

Definition 15 (w-admissible four-state labelling). *Let L be a four-state labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is w-admissible if and only if:*

- $L(a) = \mathit{IN} \implies$
  $(\forall b \in a^{-}.L(b) \in \{\mathit{OUT}, \mathit{DC}\} \wedge L(b) = \mathit{OUT} \implies w_{b^{-}\downarrow_{\mathit{IN}}} \leq_{\mathbb{S}} w_{b^{+}\downarrow_{\mathit{IN}}})$
- $L(a) = \mathit{OUT} \iff w_{a^{-}\downarrow_{\mathit{IN}}} <_{\mathbb{S}} \top$

The condition $w_{b^{-}\downarrow_{\mathtt{IN}}} \leq_{\mathbb{S}} w_{b^{+}\downarrow_{\mathtt{IN}}}$ for `IN` arguments makes sure that defenders of a are stronger than the attack of b. For an argument to be `OUT`, then, we require $w_{a^{-}\downarrow_{\mathtt{IN}}} <_{\mathbb{S}} \top$, meaning that there must exist at least an attack coming from an `IN` argument. The two labellings in Figure 5 represent w-admissible four-state labellings for the considered WAF$_{\mathbb{S}}$.

Definition 16 (w-complete four-state labelling). *Let L be a four-state labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is w-complete if and only if:*

- $L(a) = \texttt{IN} \iff$
  $(\forall b \in a^-.L(b) \in \{\texttt{OUT}, \texttt{DC}\} \wedge L(b) = \texttt{OUT} \implies w_{b^- \downarrow_{\texttt{IN}}} \leq_{\mathbb{S}} w_{b^+ \downarrow_{\texttt{IN}}})$
- $L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_{\mathbb{S}} \top$

A w-complete four-state labelling is also w-admissible. The difference is in the condition for IN arguments, which needs to be both necessary and sufficient. The four-state labellings in Figure 5 are not w-complete, since both have an UNDEC argument (e and d, respectively) which is only attacked by an OUT one.

Definition 17 (w-stable four-state labelling). *Let L be a four-state labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$. L is w-stable if and only if*

- *L is a w-complete four-state labelling and*
- *$\mathcal{A} \downarrow_{\texttt{DK}} = \emptyset$*

In contrast with the semantics in Definitions 15 and 16, a w-stable four-state labelling might not exist for a certain WAF$_{\mathbb{S}}$, depending on the presence of DK arguments. It is easy to verify that none of the labellings in Figure 5 is w-stable. We next present w-preferred and w-grounded four-state labelling for WAF$_{\mathbb{S}}$, which rely on the cardinality of the set of acceptable arguments.

Definition 18 (w-preferred labelling). *Let L be a four-state labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$. L is w-preferred if and only if*

- *L is a w-admissible four-state labelling and*
- *$\mathcal{A} \downarrow_{\texttt{IN}}$ is maximal among all the w-admissible four-state labellings*

Definition 19 (w-grounded four-state labelling). *Let L be a labelling of a WAF$_{\mathbb{S}}$ $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ and $a \in \mathcal{A}$. L is w-grounded if and only if:*

- *$L(a) = \texttt{IN} \iff$ for all w-complete four-state labellings L', $L'(a) = \texttt{IN}$ and*
- *$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_{\mathbb{S}} \top$*

We summarize in Table 1 the conditions given for the presented labellings. Next, we show how four-state labelling-based semantics for WAF$_{\mathbb{S}}$ can be traced to their three-state counterparts.

Theorem 1. *L is a w-conflict-free four-state labelling on $F = \langle \mathcal{A}, \mathcal{R} \rangle$ if and only if $L \downarrow_{\mathcal{A}}$ is a w-conflict-free three-state labelling and there exists a label renaming function such that, for all $a \in \mathcal{A}$, $(L(a) = \texttt{DC} \vee L(a) = \texttt{DK}) \implies L(a) = \texttt{UNDEC}$ and $L(a) = \texttt{UNDEC} \implies L(a) = \texttt{DC}$.*

Theorem 2. *L is a w-admissible (w-complete, w-stable, w-preferred, w-grounded) four-state labelling on $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$ with $\mathcal{A} \downarrow_{\texttt{DC}} = \emptyset$ if and only if $L \downarrow_{\mathcal{A}}$ is a w-admissible (w-complete, w-stable, w-preferred, w-grounded, respectively) three-state labelling and there exists a label renaming function such that, for all $a \in \mathcal{A}$, $L(a) = \texttt{DK} \iff L(a) = \texttt{UNDEC}$.*

Table 1
Summary of the labellings for $\text{WAF}_\mathbb{S}$.

Sem.	Conditions on IN arguments	Conditions on OUT arguments	Other
w-cf	$L(a) = \texttt{IN} \implies a^- \downarrow_{\texttt{IN}} = \emptyset$	$L(a) = \texttt{OUT} \implies a^- \downarrow_{\texttt{IN}} \neq \emptyset$	
w-adm	$L(a) = \texttt{IN} \implies a^- = a^- \downarrow_{\{\texttt{OUT},\texttt{DC}\}}$ $\wedge \forall b \in a^- \downarrow_{\texttt{OUT}} . \, w_{b^- \downarrow_{\texttt{IN}}} \leq_\mathbb{S} w_{b^+ \downarrow_{\texttt{IN}}}$	$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_\mathbb{S} \top$	
w-com	$L(a) = \texttt{IN} \iff a^- = a^- \downarrow_{\{\texttt{OUT},\texttt{DC}\}}$ $\wedge \forall b \in a^- \downarrow_{\texttt{OUT}} . \, w_{b^- \downarrow_{\texttt{IN}}} \leq_\mathbb{S} w_{b^+ \downarrow_{\texttt{IN}}}$	$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_\mathbb{S} \top$	
w-stb	$L(a) = \texttt{IN} \iff a^- = a^- \downarrow_{\{\texttt{OUT},\texttt{DC}\}}$ $\wedge \forall b \in a^- . \, w_{b^- \downarrow_{\texttt{IN}}} \leq_\mathbb{S} w_{b^+ \downarrow_{\texttt{IN}}}$	$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_\mathbb{S} \top$	$\mathcal{A} \downarrow_{\texttt{DK}} = \emptyset$
w-pre	$L(a) = \texttt{IN} \implies a^- = a^- \downarrow_{\{\texttt{OUT},\texttt{DC}\}}$ $\wedge \forall b \in a^- . \, w_{b^- \downarrow_{\texttt{IN}}} \leq_\mathbb{S} w_{b^+ \downarrow_{\texttt{IN}}}$	$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_\mathbb{S} \top$	$\mathcal{A} \downarrow_{\texttt{IN}}$ is max w-adm
w-gde	$L(a) = \texttt{IN} \iff \forall L' w\text{-com}. \; L'(a) = \texttt{IN}$	$L(a) = \texttt{OUT} \iff w_{a^- \downarrow_{\texttt{IN}}} <_\mathbb{S} \top$	

The intuition behind Theorem 2 is that the acceptability of all labelled arguments in a $\text{WAF}_\mathbb{S}$ (that is, those labelled by $L \downarrow_\mathcal{A}$) must depend only on the state of arguments that are not ignored. The proof is carried out by comparing Definition 13 with the conditions given for the four-state case. Moreover, since the four-state labelling introduced in this paper generalises the three-state one [14], we obtain a direct correspondence with weighted extensions.

Theorem 3. *Let L^F be a four-state labelling on $F = \langle \mathcal{A}, \mathcal{R}, W, \mathbb{S} \rangle$. L^F is a w-conflict-free labelling if and only if $\mathcal{A} \downarrow_{IN}$ is a w-conflict-free extension of F. Moreover, L^F is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded) four-state labelling if and only if $\mathcal{A} \downarrow_{IN}$ is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded) extension of $F' = \langle \mathcal{A} \downarrow_{\{IN, OUT, DK\}}, \mathcal{R} \downarrow_{\{IN.OUT, DK\}} \rangle$.*

Finally, we observe that any four-state weighted labelling instantiated with a boolean c-semiring corresponds to a four-state labelling. Indeed, when a $\text{WAF}_\mathbb{S}$ is instantiated with a boolean c-semiring, all the attacks in the framework are associated with the value $false$ and $w_{a^- \downarrow_{\texttt{IN}}}$ always corresponds to $false$ if a has at least one attacker.

Theorem 4. *Let F be a $\text{WAF}_\mathbb{S}$ where $\mathbb{S}$ is a boolean c-semiring. If L is a w-admissible (respectively w-complete, w-stable, w-preferred, w-grounded) four-state labelling of F, then L is also an admissible (respectively complete, stable, preferred, grounded) four-state labelling.*

4. Related Work

The problem of extending classical AFs with values expressing the strength of arguments and attacks is widely studied, and many different approaches have been presented in the literature. Amgoud and Cayrol [25] take into account preference orderings for comparing arguments, while in a paper by Bench-Capon [26] the success of an attack conducted by an argument toward another one depends on an ordering among the "values" promoted by each argument.

A study on bipolar Weighted AFs is conducted by Pazienza et al. [27], who present an extension for weighted frameworks taking into account two different types of relations: one for attack and one for support. We consider, instead, Weighted AFs with only one type of possible relation between arguments (the attack relation). Note that there exist techniques for translating bipolar AFs into classical AFs [28], although the weighted case has not been investigated yet. Another formalism based on a notion of strength is given in a paper by Baroni et al. [29], where arguments in Quantitative Argumentation Debate Frameworks are evaluated through a scoring system. The main difference with our work lies in the fact that we take into account the basic definition of Weighted AFs [18], without further refinements on the framework level. Moreover, our study is focused on the interpretation of the labelling in the weighted case.

Labelling functions using four justification states are proposed by various authors [22, 6, 23]; the additional label identifies those arguments that should not be considered during the computation of acceptability. A more general labelling has also been proposed [13], which unifies different representations and can be mapped into sets of extensions. However, weights are not considered in any of these works.

For what concern the notion of weighted defence, many possible definitions can be considered: for instance, Martìnez et al. [10] use the relative strength of the attacks in order to determine if some defence constraints are satisfied, while Coste-Marquis et al. [9] aggregate the weights of the defence and check if this value is greater than the weight of the corresponding attack. On the other hand, we exploited the notion of collective weighted defence [7], which also generalises the other two approaches mentioned above.

5. Conclusion and Future Work

In this paper, we introduce labelling for Weighted AFs that uses up to four states to discern various grades of acceptability for arguments, namely IN, OUT, DK and DC. We also identify sets of conditions under which the proposed labelling corresponds to a weighted extension for some semantics. Our labelling function generalises both the classical approach for the non-weighted case and the three-state labelling for $WAF_{\mathbb{S}}$.

The work can be expanded in many directions. In our setting, arguments only attacked by DC arguments are always labelled IN. Is future work, we want to consider a pessimistic interpretation for ignored arguments: since a DC-labelled argument a could be (re)considered into the AF, thus gaining an IN, OUT or DK label, arguments only attacked by a could be labelled OUT in turn. The definition we give of a four-state labelling-based semantics for Weighted AFs does not include conditions for DK arguments, since they are indirectly obtained from IN and OUT. In this sense, we would like to investigate the possible advantages of giving explicit conditions for labelling the DK arguments, similarly to what is by Modgil and Caminada [30] for classical AFs. We also plan to consider w-strongly admissible extensions [31, 14] and introduce the respective four-state labelling. In addition to the collective weighted defence [7] that we used in this paper, there are other notions of weighted defence [9, 10] that could be considered for obtaining different variations of the four-state weighted labelling. We would also like to take into account a relaxed version of the weighted defence [8] where two parameters (α and γ) are used to both enable a tolerance threshold for inconsistencies inside extensions and consider arguments that are not fully w-defended. Finally, extended versions of AFs (e.g., Bipolar Argumentation Frameworks [32]) could be investigated from the perspective of the four-state labelling-based semantics.

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